

Numerical Analysis of Mixed Convection of Hybrid Ferrofluid in a Double Lid-driven Porous Hexagonal Cavity with a Square Heated Block

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ABSTRACT: This study numerically explores magnetohydrodynamic (MHD) mixed convection (MC) of an Fe_3O_4 - Al_2O_3 /water hybrid ferrofluid (HFF) in a double lid-driven porous hexagonal cavity containing a centrally placed square heated block. The top and bottom walls move in opposite directions ($U = 1$ and $U = -1$) and are kept cold, the slanted walls are adiabatic, and a uniform horizontal magnetic field acts on the fluid under the influence of gravity. The Darcy–Brinkman–Forchheimer model describes the flow through the copper-foam porous medium, and the governing equations are solved with the Galerkin weighted residual finite element method (GWR-FEM). The unsteady flow and thermal fields are analyzed through streamlines, isotherms and the average Nusselt number (Nu_{av}) on the surface of the heated block, for Richardson numbers $Ri = 0.1, 1$ and 10 , Hartmann numbers $Ha = 0, 20$ and 40 , $Re = 100$ and $Da = 10^{-2}$. The results show that at low Ri the flow is dominated by shear from the moving lids, while at high Ri buoyancy produces a strong circulation around the block, thinner thermal boundary layers and a higher Nu_{av} . In contrast, increasing Ha damps the fluid motion through the Lorentz force, weakens the vortices, moves the thermal field toward conduction and lowers Nu_{av} . In all cases, Nu_{av} is highest at the start of the simulation and then decreases toward a quasi-steady value. The findings are useful for the thermal management of electronic components, heat transfers (HT) and magnetically controlled cooling systems.

1. Introduction

Excessive heat generated during operation limits the performance and lifetime of modern devices such as processors, micro- and nano-electronic systems, vehicles and

reactors. In fact, overheating may even result in failure in severe conditions, which necessitates the application of suitable HT approaches for removal of the extra heat [1]. The lid-driven cavity flow system has been traditionally used for the investigation of MC, where the flows induced due to forced and natural causes compete. Earlier work uses square or rectangular cavities. Non-rectangular enclosures such as trapezoidal, L-shaped and hexagonal cavities are more realistic for engineering equipment, because their inclined walls and corners introduce secondary vortices, multi-cell structures and non-uniform temperature gradients [2,3]. The hexagonal shape appears in compact heat exchangers, solar collectors, honeycomb-type cooling structures and electronic enclosures, yet reports on MC in hexagonal cavities remain scarce.

Porous media change the flow and HT inside an enclosure considerably. The Darcy–Brinkman–Forschheimer model represents Darcy drag, viscous diffusion and inertial resistance in one framework and is widely used for engineering porous structures [4,5]. Further research by Khanafer et al. [6] demonstrated buoyancy-driven HT enhancement in two-dimensional enclosures using nanofluids, while Abu-Nada et al. [7] analyzed the influence of variable nanofluid properties on natural convection dynamics.

In recent years, attention has shifted toward multi-component and porous configurations to achieve superior thermal performance. For example, Pramanik et al. [8] did a computational study on HFF in a porous L-cavity that contains an inner cylinder, which brings out the significance of porous medium and magnetism in controlling flow convective patterns. Porous medium is responsible for vortex development, heat conduction, and dominance of convection, and metals like the copper foam increase effective thermal conductivity [9].

Hybrid nanofluids (HNFs) are high-performance working fluids with superior thermophysical characteristics and are extensively used for the augmentation of HT in compact thermal systems. Working fluids using nanotechnology have paved yet another way for enhancing HT. Hybrid nanofluids, containing two or more types of nanoparticles, possess higher thermal conductivity, superior stability, and heat transfer characteristics when compared to the conventional nanofluids [10]. $\text{Fe}_3\text{O}_4\text{-Al}_2\text{O}_3/\text{Water}$ is an interesting combination because of the magnetic property of Fe_3O_4 and superior stability and conductivity in the presence of Al_2O_3 [11,12]. Because of its magnetic and conductive nature, Lorentz force will act against the flow, and,

therefore, Hartmann number can be considered as an excellent passive control parameter for the flow and HT [13,14].

The presence of a heat-producing object brings additional complications to physics. The blocks producing heat are commonly used to represent electronic chips or other similar equipment parts and significantly affect streamlines' development and buoyancy flows [13,15]. Double lid-driven configurations, in which opposite walls move in opposite directions, add controllable shear that can be used to tune HT [16]. Although MC with heated blocks has been studied in square and trapezoidal cavities, the combined effects of a double lid-driven hexagonal geometry, a porous medium, a HFF and a magnetic field have not been investigated together.

Finite element methods handle curved or inclined boundaries and unstructured meshes more naturally than finite difference methods, and GWR-FEM has been shown to be reliable for MC, nanofluid and MHD problems in complex domains [15, 17]. The present work therefore simulates MHD MC of an Fe_3O_4 – Al_2O_3 /water HFF in a double lid-driven porous hexagonal cavity with a square heated block using GWRFEM. The main objectives are to (i) assess the effect of the Richardson number on the flow and thermal fields, (ii) assess the effect of the Hartmann number, and (iii) quantify the resulting HT rate through Nu_{av} on the heated block.

2. Mathematical Formulation

Figure 1 shows the physical model and coordinate system. A two-dimensional hexagonal cavity of characteristic length H is filled with copper foam (40 PPI) saturated with Fe_3O_4 – Al_2O_3 /water HFF. A square block at the center is maintained at the hot temperature Th , with internal heat generation q_R considered. The top wall moves with dimensionless velocity $U = 1$ and the bottom wall with $U = -1$, both at the cold temperature T_c . The four slanted walls are stationary and adiabatic. A uniform magnetic field B_0 acts in the horizontal direction and gravity g acts downward. The flow is assumed to be laminar, incompressible and unsteady. The fluid and the porous matrix are in local thermal equilibrium, and the thermophysical properties are constant except for the density in the buoyancy term, which follows the Boussinesq approximation. The properties of the nanoparticles, the base fluid and the copper foam are given in Table 1.

The physical properties of this HFF remain constant, except for the density variation in the body force component of the momentum equation, which is attributed to the

Boussinesq approximation. Based on these premises, the governing equations without dimensions are: (Nield and Bejan [5], & Pramanik et. al [18])

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial U}{\partial \tau} + \frac{1}{\varepsilon^2} \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = & -\frac{\partial P}{\partial X} + \frac{1}{\varepsilon} \frac{v_{hff}}{v_{ff}} \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + \frac{\beta_{hff}}{\beta_{ff}} \frac{Ra}{PrRe} \theta \sin \phi \\ & - \frac{v_{hff}}{v_{ff}} \frac{1}{ReDa} U - \frac{F_c}{\sqrt{Da}} U \sqrt{U^2 + V^2} \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\partial V}{\partial \tau} + \frac{1}{\varepsilon^2} \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = & -\frac{\partial P}{\partial Y} + \frac{1}{\varepsilon} \frac{v_{hff}}{v_{ff}} \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\beta_{hff}}{\beta_{ff}} \frac{Ra}{PrRe} \theta \cos \phi - \\ & \frac{v_{hff}}{v_{ff}} \frac{V}{ReDa} - \frac{F_c V \sqrt{U^2 + V^2}}{\sqrt{Da}} - \frac{Ha^2}{Re} \frac{\sigma_{hff}}{\sigma_{ff}} \frac{\rho_{hff}}{\rho_{ff}} V \end{aligned} \quad (3)$$

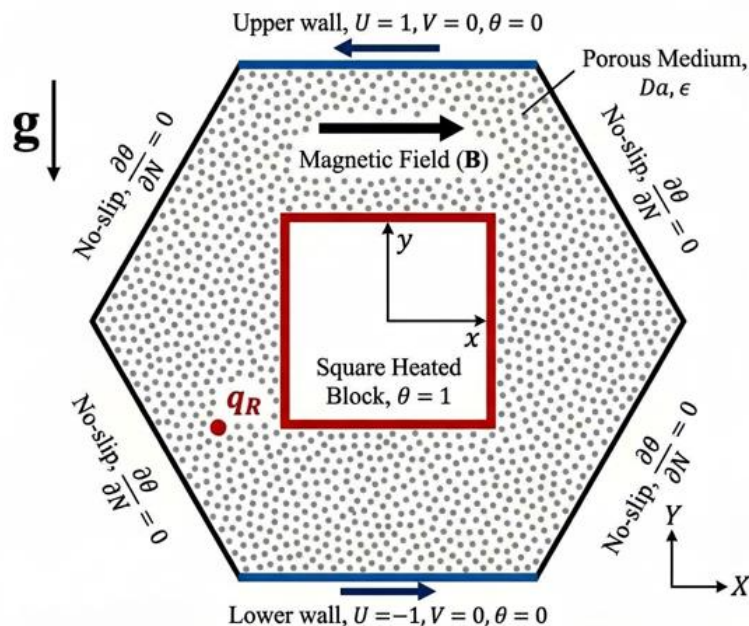


Fig. 1. Schematic model and coordinate system.

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{eff,hff}}{\alpha_{ff}} \frac{1}{PrRe} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \frac{(\rho c_p)_{ff}}{\varepsilon(\rho c_p)_{ff}} q_R \quad (4)$$

Here Prandtl number $Pr = \frac{\nu_{ff}}{\alpha_{ff}}$, Reynolds number $Re = \frac{U_0 H}{\nu_{ff}}$, Darcy number $Da = \frac{K}{H^2}$, Rayleigh number $Ra = \frac{g \beta_{ff} (T_h - T_c) H^3}{\nu_{ff} \alpha_{ff}}$, Hartmann number $Ha = B_0 H \sqrt{\frac{\sigma_{ff}}{\mu_{ff}}}$, and $q_R = \frac{q_0}{(\rho c_p)_{ff} (T_h - T_c) U_0}$ is heat generation.

These are the boundary conditions that are compatible with the governing equations shown earlier:

$$\begin{array}{l} \text{At the top wall of the cavity: } U = 1, V = 0, \theta = 0 \\ \text{At the bottom wall of the cavity: } U = -1, V = 0, \theta = 0 \end{array} \quad \left| \quad \begin{array}{l} \text{Rest of walls of the cavity: } U = V = 0, \frac{\partial \theta}{\partial X} = 0 \\ \text{All walls of the square of the block: } U = V = 0, \theta = 1 \end{array} \right.$$

The effective thermal diffusion of the hybrid $\alpha_{eff,hnf}$ and base fluid $\alpha_{eff,ff}$ of porous medium is shown below.

$$\alpha_{eff,hnf} = \frac{\kappa_{eff,hff}}{(\rho c_p)_{hff}} \text{ and } \alpha_{eff,ff} = \frac{\kappa_{eff,bf}}{(\rho c_p)_{ff}} \quad (5)$$

For base fluid $\kappa_{eff,bf}$ and hybrid fluid $\kappa_{eff,hff}$ represented respectively as:

$$\kappa_{eff,bf} = \varepsilon \kappa_{bf} + (1 - \varepsilon) \kappa_s \text{ and } \kappa_{eff,hff} = \varepsilon \kappa_{hff} + (1 - \varepsilon) \kappa_s \quad (6)$$

The following are listed as the HNFs physical characteristics:

Following are the heat capacity [14, 19], effective dynamic viscosity [20], thermal expansion and thermal conductivity [21], the effective density [22, 23] and thermal diffusivity [24, 25] for both the nanofluid and the HNF:

$$\alpha_{hff} = \frac{\kappa_{hff}}{(\rho c_p)_{hff}} \quad (7)$$

$$\rho_{hff} = \delta_{Al_2O_3} \rho_{Al_2O_3} + \delta_{Cu} \rho_{Cu} + (1 - \delta) \rho_{ff} \quad (8)$$

$$\delta = \delta_{Al_2O_3} + \delta_{Fe_3O_4} \quad (9)$$

$$\mu_{hff} = \frac{\mu_{bf}}{\left(1 - (\delta_{Al_2O_3} + \delta_{Cu})\right)^{2.5}} \quad (10)$$

$$(\rho C_p)_{hmf} = \delta_{Al_2O_3}(\rho C_p)_{Al_2O_3} + \delta_{Cu}(\rho C_p)_{Cu} + (1 - \delta)(\rho C_p)_{Fe_3O_4} \quad (11)$$

$$(\rho\beta)_{hmf} = \delta_{Al_2O_3}(\rho\beta)_{Al_2O_3} + \delta_{Cu}(\rho\beta)_{Cu} + (1 - \delta)(\rho\beta)_{bf} \quad (12)$$

$$\frac{\kappa_{hff}}{\kappa_{ff}} = \left(\frac{(\delta_{Al_2O_3}\kappa_{Al_2O_3} + \delta_{Cu}\kappa_{Cu})}{\delta} + 2\kappa_{Fe_3O_4} \right) \times \left(\frac{(\delta_{Al_2O_3}\kappa_{Al_2O_3} + \delta_{Cu}\kappa_{Cu})}{\delta} + 2\kappa_{Fe_3O_4} \right)^{-1} \\ + \frac{(\rho C_p)_{hff}}{2} \delta \sqrt{\frac{2K_b T_{Ref}}{3\pi\mu_{hff}d_p}} \quad (13)$$

$$\frac{\sigma_{hff}}{\sigma_{ff}} = 1 + \left(\frac{\left(\frac{3(\delta_{Al_2O_3}\kappa_{Al_2O_3} + \delta_{Cu}\kappa_{Cu})}{\sigma_{bf}} - (\delta_{Al_2O_3} + \delta_{Cu}) \right)}{\left(\frac{(\delta_{Al_2O_3}\kappa_{Al_2O_3} + \delta_{Cu}\kappa_{Cu})}{\delta\sigma_{Fe_3O_4}} + 2 \right) - \left(\frac{(\delta_{Al_2O_3}\kappa_{Al_2O_3} + \delta_{Cu}\kappa_{Cu})}{\sigma_{Fe_3O_4}} - (\delta_{Al_2O_3} + \delta_{Cu}) \right)} \right) \quad (14)$$

The Nu_{av} of the enclosure's horizontally heated surface can be expressed as follows:

$$Nu_{ave} = -\frac{\kappa_{hff}}{\kappa_{ff}} \int_0^H \frac{\partial \theta}{\partial Y} dX \text{ to the horizontally heated wall.}$$

Where κ_{bf} is the thermal conductivity of the base fluid, T_{ref} is reference temperature, F_c Forcheimer constant, $K_B = 1.380649 \times 10^{-23}$ is the Boltzman constant and $d_p = 10\text{nm}$ is the size of the particle.

Table 1. Thermophysical properties of Al_2O_3 , Fe_3O_4 [11, 26] and Copper foam (40PPI) [9]

Physical Properties	Unit	Nanoparticles			
		Fe_3O_4	Al_2O_3	H_2O	Copper Foam (40PPI)
C_p	J/kgK	600	765	4179	385
ρ	kg/m ³	1200	3970	997.1	9095
k	W/mK	401	40	0.613	401
σ	simens/m	5.96×10^7	1×10^{-10}	5.5×10^{-6}	---
$\beta \times 10^5$	K^{-1}	2.67	0.85	21	---
ε		---	---		0.8981
K		---	---		1.28355×10^{-7}

3. Computational Procedure

The dimensionless equations (1) – (4), together with the boundary conditions, are solved with GWR-FEM [17]. The hexagonal domain is divided into unstructured triangular elements, with mesh refinement concentrated near the square block, the moving lids and the corners of the hexagon, as is standard for cavity-driven flows [27]. Quadratic (six-node) interpolation is used for velocity and temperature and linear interpolation for pressure to satisfy the convergence condition. The weight function is applied to each governing equation and then integrates over the domain, and the diffusion terms are integrated through integration by parts to formulate the weak form. The Darcy term, Forchheimer equations and Lorentz force are incorporated directly into the momentum weak form. The Backward Euler scheme is used for time integration while the nonlinear convective and Forchheimer terms are linearized using Newton iterations. The thermophysical properties of the ferrofluid are calculated in every iteration through the mixture model correlations stated earlier. Convergence is reached if the relative errors in the velocities, pressures and temperatures are less than 10^{-6} . After convergence, streamlines, isotherms and Nu_{av} are computed.

4. Test for Grid Refinement

A grid-independence test was carried out to ensure that the numerical results do not depend on the mesh size. The hexagonal domain was discretized with unstructured triangular quadratic elements, as shown in Figure 2. The mesh is refined near the surface of the square heated block, where the temperature and velocity gradients are steepest, and it is coarser toward the outer walls. The number of elements increased progressively over five refinement levels, from 1013 to 7570 elements. Table 2 lists the average Nusselt number (Nu_{av}) on the surface of the heated block at $Ri = 1$, $Ha = 0$ and $Re = 100$.

Nu_{av} increases from 10.5051 for the coarsest mesh (1013 elements) to 12.4057 for 2460 elements, a change of 15%. The change falls to 9% for 4790 elements and to 0.6% for 6585 elements ($Nu_{av} = 13.7078$). Between 6585 and 7570 elements, Nu_{av} changes by only 0.13% (13.7263), so the solution is effectively independent of the mesh. The mesh with 6585 elements, where the change is already below 1%, was therefore adopted for all computations as the best compromise between accuracy and computational cost.

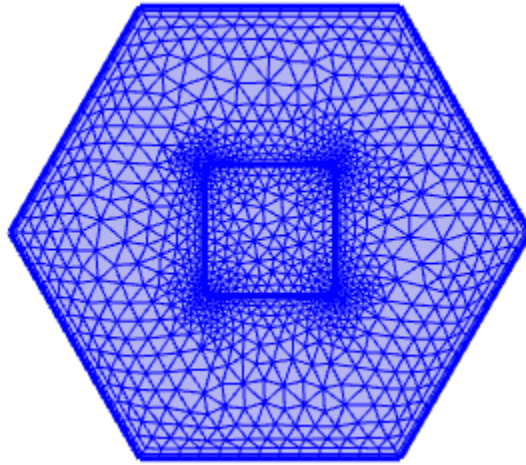


Fig. 2. Standard Finite Element discretizing a Hexagonal Cavity

Table 2. Nu_{ave} for different mesh sizes.

Refinements	No. of elements	Nu_{ave}	% Error
1 st	1013 (Coarser)	10.5051	
2 nd	2460 (Coarse)	12.4057	15%
3 rd	4790 (Normal)	13.621	9%
4 th	6585 (Fine)	13.7078	0.6%
5 th	7570 (Finer)	13.7263	0.13%

5. Results and Discussion

The effects of the Richardson number (Ri) and the Hartmann number (Ha) on the flow and temperature fields are examined for $Re = 100$, a fixed Darcy number $Da = 10^{-2}$, $q_R = 1$, and solid volume fraction $\sigma = 0.4$. Results are presented as streamlines, isotherms and Nu_{av} on the square heated block.

5.1 Effect of Richardson Numbers (Ri)

Figure 3 shows the streamlines for $Ri = 0.1, 1$ and 10 with $Re = 100$ and fixed Ha and Da at $\tau = 0.2, 0.4$ and 0.6 . At $Ri = 0.1$, forced convection dominates. Fast flow stays in thin layers along the moving lids, circulation near the block is weak, and the pattern changes little with time. At $Ri = 1$, buoyancy and shear are comparable. Asymmetric vortices form on either side of the block and grow with time. At $Ri = 10$, buoyancy dominates. One strong circulation wraps around the heated block, with the highest velocities beside its vertical sides, and it expands toward the walls as time advances.

Increasing Ri therefore strengthens the flow around the block, which should raise the HT rate.

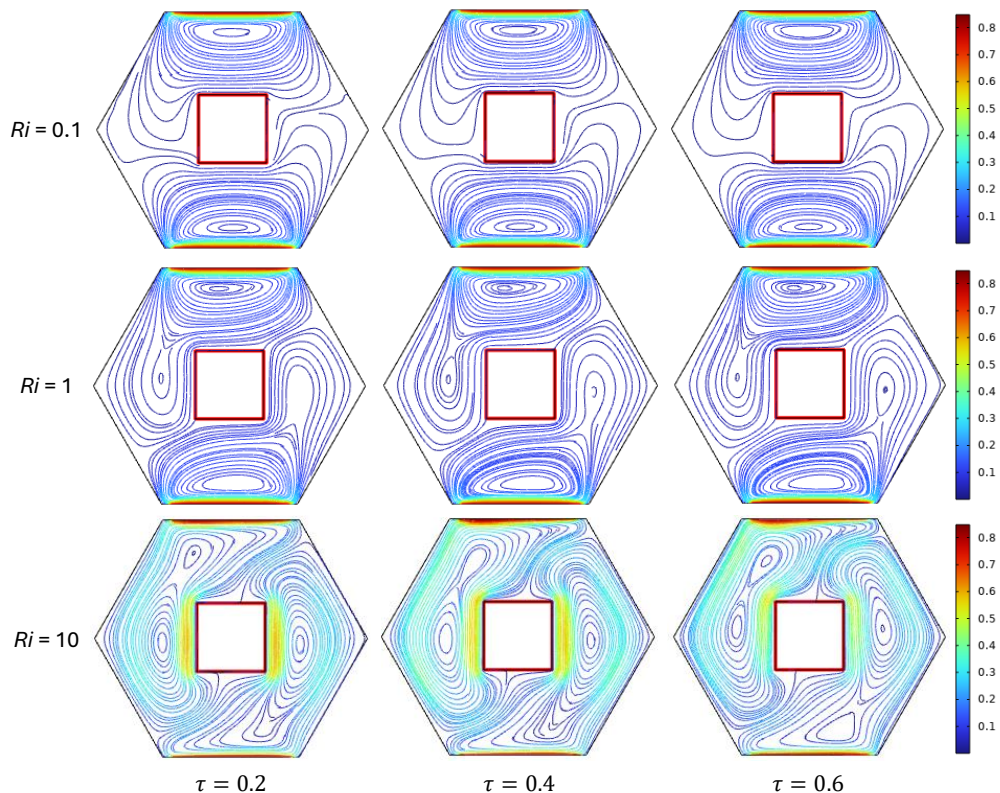


Fig. 3. Streamlines for various Ri with various Time at $Re = 100$ and fixed Ha and Da .

Figure 4 shows the isotherms for $Ri = 0.1, 1$ and 10 at $\tau = 0.2, 0.4$ and 0.6 . For $Ri = 0.1$, the isolines are densely packed around the moving walls. The area around the block remains nearly void of the isolines, which do not vary much with time; thus, heat transfer occurs purely through conduction. For $Ri = 1$, the isolines tend to migrate towards the block and become asymmetrical around the vertical surfaces of the block, thereby demonstrating the commencement of buoyancy-driven heat transfer.

For $Ri = 10$, thick isolines surround the block, extending vertically down the side surfaces of the block. The thermal boundary layer gets thinner, the temperature gradients become steeper, and the thermal domain expands towards the wall with time.

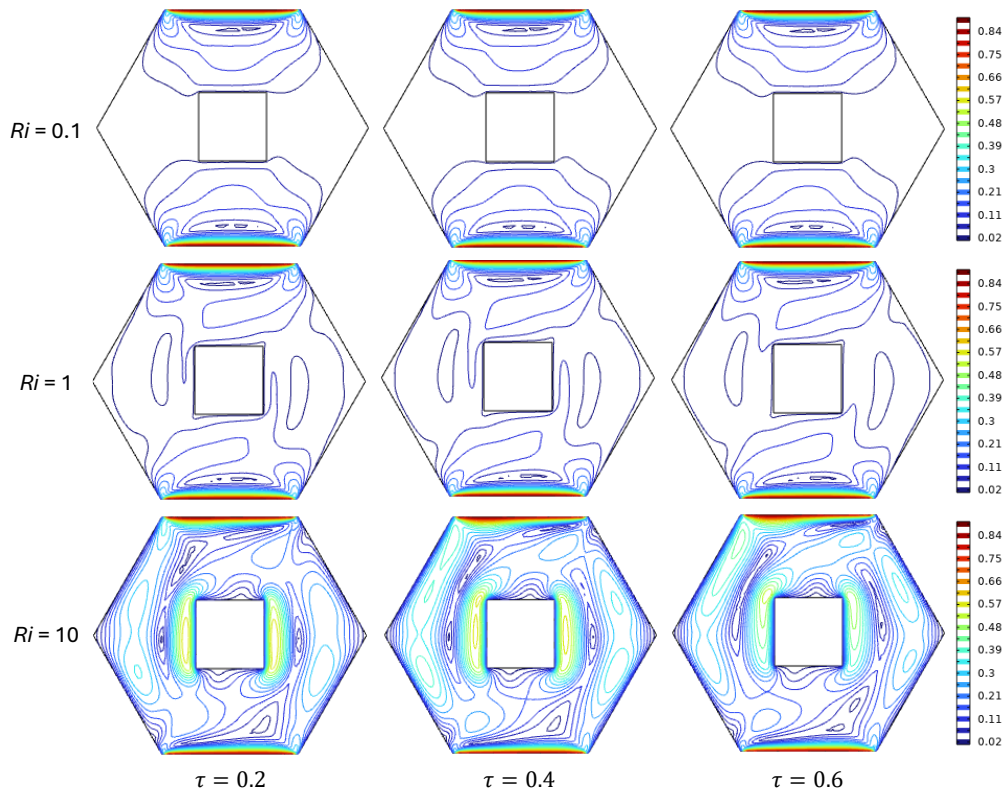


Fig. 4. Isotherms for various Ri with various Time at $Re = 100$ and fixed Ha and Da .

Figure 5 shows Nu_{av} on the heated block against time for $Ri = 0.1, 1$ and 10 . For all cases, Nu_{av} starts near 14 and drops sharply until $\tau \approx 0.3$, then declines slowly toward a quasi-steady value as the fluid around the block warms. At every time, Nu_{av} increases with Ri . $Ri = 1$ and $Ri = 10$ are close together, while $Ri = 0.1$ is much lower. At $\tau = 1$, Nu_{av} is about 4.2 for $Ri = 10$, 3.6 for $Ri = 1$ and 2.2 for $Ri = 0.1$. Stronger buoyancy at higher Ri drives more vigorous circulation around the block, which thins the thermal boundary layer and raises HT. This agrees with the streamlines and isotherms in Figures 3 and 4.

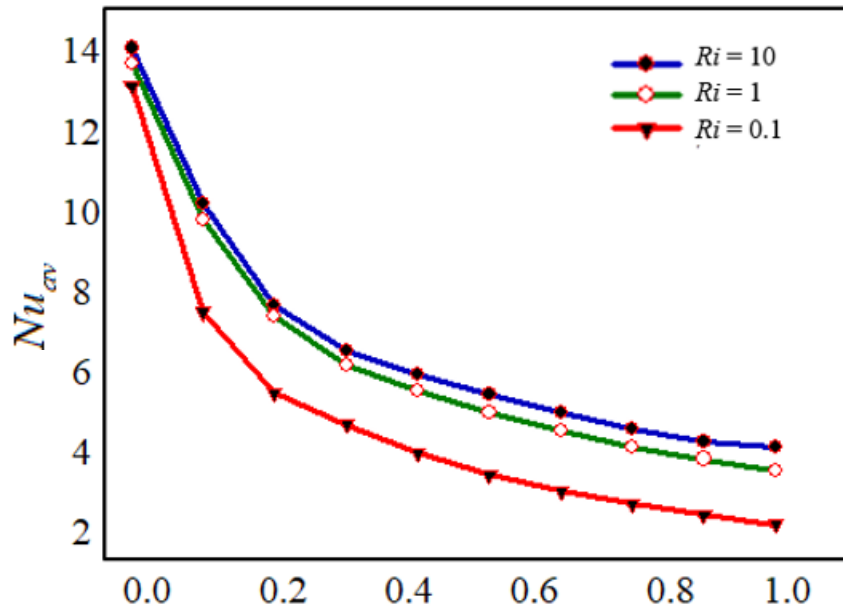


Fig. 5. The distinction of Nu_{av} with time for different Ri at $Re = 100$, fixed Ha and Da .

5.2 Effect of Hartmann Numbers (Ha)

Figure 6 shows the streamlines for $Ha = 0, 20$ and 40 at $\tau = 0.2, 0.4$ and 0.6 . Without the presence of magnetic field ($Ha = 0$), the flow intensity is the highest. Large vortices appear on the sides of the block as well as in the upper and lower parts of the cavity and they keep developing and moving with the progression of time. With the increase in Ha , the Lorentz force works against the fluid motion, whereas, for horizontal magnetic field, the vertical component of velocity of the fluid motion is dampened. The vortices on the sides of the block become smaller and weaker with their cores moving towards the walls. For $Ha = 40$, the streamlines become more horizontal and smoother, while the flow becomes much quieter around the block, being localized only to the shear layers formed at the lids' surfaces.

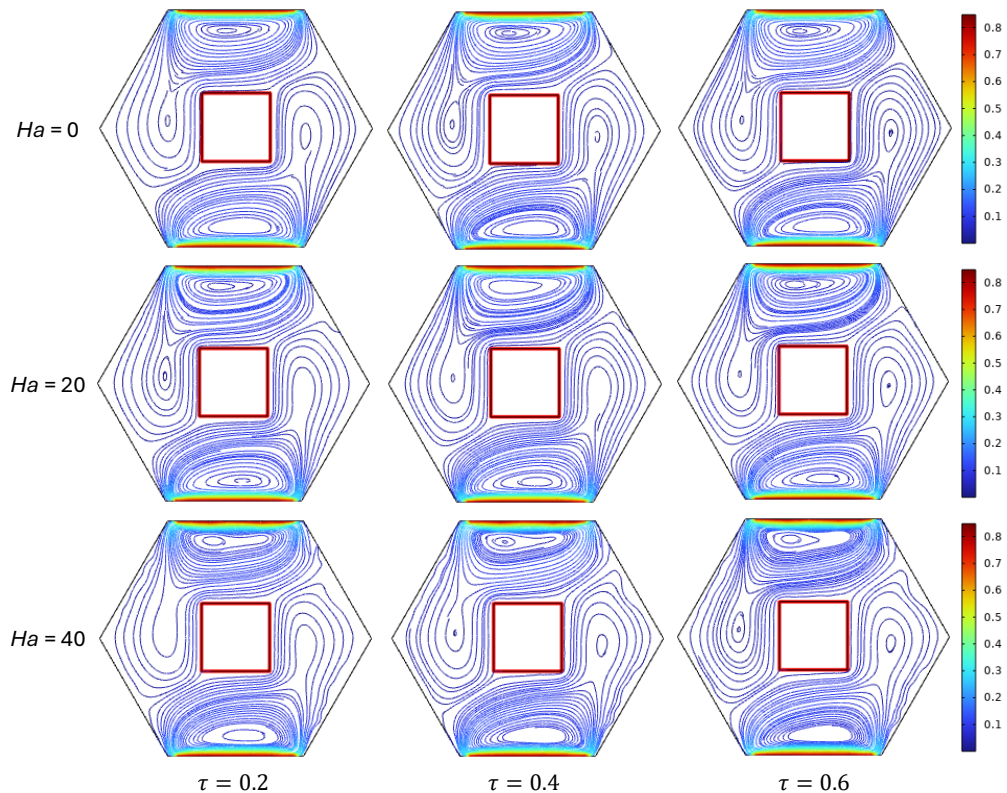


Fig. 6. Streamlines for various Ha with various Time at $Re = 100$ and fixed Ri and Da .

Figure 7 shows the isotherms for $Ha = 0, 20$ and 40 at $\tau = 0.2, 0.4$ and 0.6 . At $Ha = 0$, the contours are strongly distorted. Dense bands lie along the moving lids, and elongated contour lines curve around the block and extend into the side regions, which shows active convective transport. As Ha increases, these lines become shorter, smoother and more evenly spaced, and the pattern near the block flattens. At $Ha = 40$, the thermal field is confined to a thin region near the lids, with only weak distortion elsewhere. The time evolution is also slower at high Ha . The magnetic field therefore weakens convection, moves the thermal field toward conduction and reduces HT from the block.

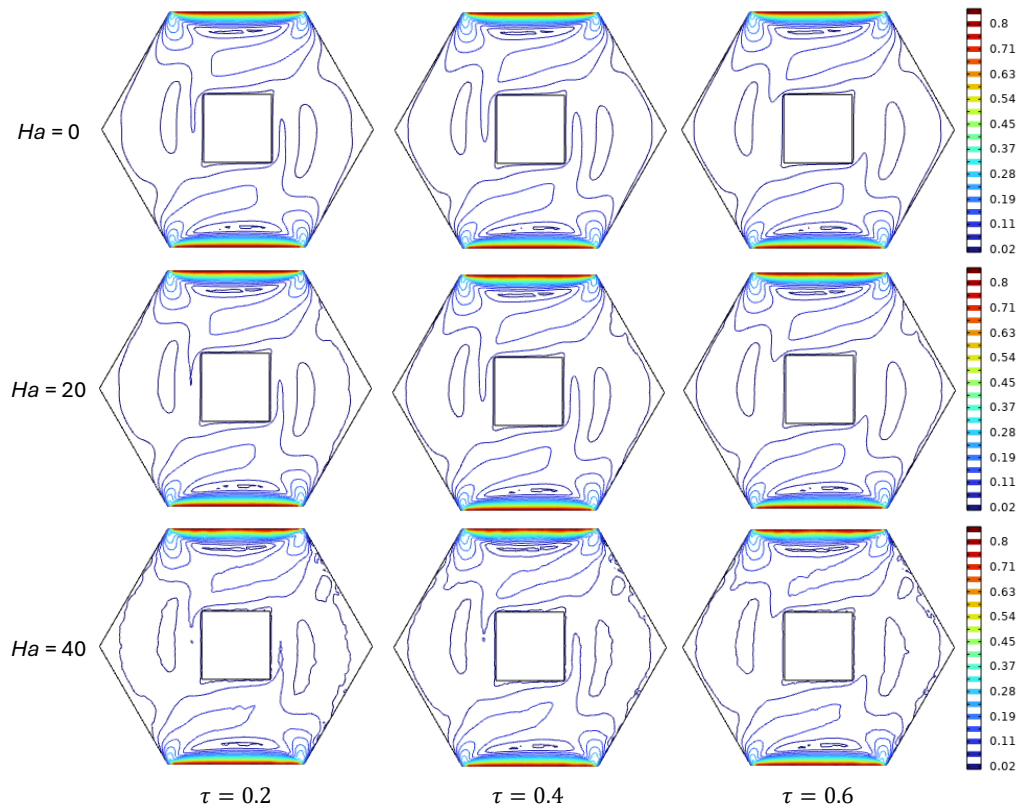


Fig. 7. Isotherms for various Ha with various Time at $Re = 100$ and fixed Ri and Da .

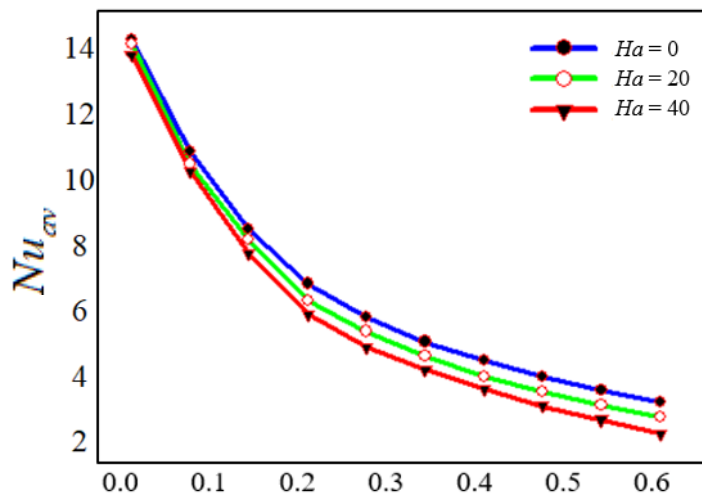


Fig. 8. The distinction of Nu_{av} for different Ha with $Re = 100$, fixed Ri and Da .

Figure 8 shows Nu_{av} on the heated block against time for $Ha = 0, 20$ and 40 . For all cases, Nu_{av} starts high (about 14) and drops quickly until $\tau \approx 0.2$, then declines slowly toward a quasi-steady value. At every time, Nu_{av} decreases as Ha increases. At $\tau \approx 0.6$, it is about 3.2 for $Ha = 0$, 2.8 for $Ha = 20$ and 2.3 for $Ha = 40$. The Lorentz force damps the flow, so convection weakens, the thermal boundary layer thickens and HT falls.

6. Conclusions

MHD mixed convection of an $Fe_3O_4-Al_2O_3$ /water HFF in a double lid-driven porous hexagonal cavity with a square heated block has been studied numerically with GWR-FEM. The main conclusions are:

- i. The Richardson number strongly changes the flow and thermal structure. At $Ri = 0.1$, shear from the moving lids dominates, with weak circulation around the block. As Ri increases, buoyancy produces asymmetric vortices ($Ri = 1$) and then a strong circulation wrapping around the heated block ($Ri = 10$), with thinner thermal boundary layers.
- ii. Heat transferred to Nu_{av} in the hot slab rises with increasing Ri throughout non-dimensional time (τ). This occurs mostly at the beginning of the computation, after which it decreases and stabilizes to a quasi-steady state.
- iii. The Hartmann number (Ha) has the opposite effect. The Lorentz force damps the fluid motion, weakens the vortices, makes the isotherms smoother and lowers Nu_{av} , so HT moves toward conduction. The reduction is small from $Ha = 0$ to 20 and larger at $Ha = 40$.

These results can help in designing compact cooling systems, heat exchangers and magnetically controlled thermal devices that use hexagonal enclosures.

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Nomenclature

Da	Darcy number	Nu_{ave}	Average Nusselt number of
C_p	Specific heat at constant pressure, J.kg.K ⁻¹		heat source
Ha	Hartmann number	P	Dimensionless pressure
κ	Thermal conductivity, Wm ⁻¹ K ⁻¹	Re	Reynolds number
K	Permeability, m ²	Ri	Richardson number
		T	Temperature (K)

T_c	Cold wall temperature, K	τ	Dimensionless time
T_h	Hot wall temperature, K	ν	Kinematic viscosity (m^2s^{-1})
U, V	Dimensionless velocity components	ρ	Density (kgm^{-3})
X, Y	Non-dimensional coordinates	θ	Dimensionless temperature
HT	Heat Transfer	δ	Solid volume fraction
MC	Mixed Convection	σ	Effective electrical conductivity
HNF	Hybrid nanofluid	Subscripts	
HFF	Hybrid ferrofluid	p	Nanoparticle
Greek symbols		bf	Base fluid
α	Thermal diffusivity (m^2s^{-1})	nf	Nanofluid
β	Thermal expansion coefficient (K^{-1})	hnf	Hybrid nanofluid

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